A Study of Rough Set Approximations for a Complementary Graph

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Abstract— The objective of the paper is to compute the definable sub graph of a complement graph G' based on the approximations of rough set using neighborhood relation. By applying the concepts of accuracy measures of approximations of subgraph H of a complementary graph , the calculation of exactness and roughness of subgraph a complementary graph are to be made. Furthermore properties of approximations are discussed and concluded.

Keywords- Rough sets, neighborhood, complement graph, lower and upper approximation, accuracy measures.

I. INTRODUCTION

Rough graph theory has emerged as a potential area of interdisciplinary research and it is of recent interest. Rough graph is the graph that can be used to research uncertainity problems and is also a new research tool to analyse the system of rough characteristics. Rough set theory and graph theory both have a variety of applications across different fields. Rough set theory is a powerful mathematics tool applied in data mining ,pattern recognition, control theory, artificial intelligence etc.,

Graphs are important discrete structures consisting of vertices and edges that connect these vertices and they describe the relationship among objects. Problems in almost every discipline can be solved using graph models. Graph theory is rapidly moving into the mainstream of mathematics mainly due to its applications in diverse fields including biochemistry(ie.,geonomics),electrical engineering (communicatin networks and coding),computer science(algoirithm and computations) and operations research(ie.,scheduling) etc.,
There are numerous applications concerning connections by rough sets and graph such as job assignment ,chemical classification,etc..

Motivated mainly by the work in [5],[6],[10], the present work is established for complementary graph using the concepts of lower and upper approximation in rough set theory.

The current work is to determine the definable subgraph H of a complementary graph based on the approximations of rough set using neighborhood relation.By applying the concepts of accuracy measures of approximations of subgraph H of a complementary graph ,the calculations of exactness and roughness are to be made.In the last section properties of approximation are discussed.

II .PRELIMINARIES

A graph G is an ordered triple consists of a vertex set V(G),an edge set E(G),and a relation that associates with each edge of G an unordered pair of vertices (not necessarily distinct) called its end points. A graph is said to be finite if the set V(G) is finite otherwise it is infinite. The pair of vertices are connected by an edge are called adjacent vertices. A vertex of a graph which is not adjacent to any other vertex is called isolated vertex. An edge of a graph that joins a vertex to itself is called a loop. In a directed or undirected graph, certain pairs of vertices are joined by more than one edge, such edges are called parallel edges. A simple graph is a graph having no loops or multiple edges. In a graph G = (V, E), each edge e in E is associated with an ordered pair of vertices then G is called a directed graph. yersaThe complement G' of a simple graph G is a graph with vertex set V(G) defined by uv $\in$ E(G') if and only if uv $\notin$ E(G)[1],[4],[9] .A definable graph is a set X of vertices together with a set E $\subset$ $[X]^2$ of edges .Equivalently a graph is a set X together with a symmetric , irreflexive relation R ,the edge relation. We usually describe a graph in terms of the set E of edges rather than the edge edge relation R.

The theory of rough sets proposed by Pawlak [7] is an extension of set theory for the study of intelligent systems characterized by insufficient systems characterized by insufficient and incomplete information. In Pawlak space a subset A$\subseteq$ X has two possibilities ,rough or exact.For any subgraph H of a graph G H is exact subgraph of G if and only if Bd V(H) = φ,otherwise rough. Rough set concept can be defined quite generally by means of topological operations, interior and closure called approximations [7]. Roughness as opposed to accuracy represents degree of incompleteness of knowledge R about the set X. In clear the accuracy coefficient expresses how large the boundary region of the set is, not the structure of the boundary.

In [7], the R-lower, R-upper approximation and R – boundary of X is defined as ,

$$\underline{R}\ X = \bigcup\{y \in U / R:\ y \subseteq X\}$$

$$\overline{R}\ X = \bigcup\{y \in U / R:\ y \cap X \neq \phi\}$$

$$BN_R(X) = \underline{R}X - \overline{R}X.$$

Then (i) X is R–definable if and only if

$$\overline{R}X = \underline{R}X$$

(ii) X is rough with respect to R if and only if $\overline{R}X \neq \underline{R}X$.

Rough sets approximations have the following properties:

1) $\underline{R}(X) \subseteq X \subseteq \overline{R}(X)$

2) $\underline{R}(\varphi) = \overline{R}(X) = \varphi$; $\underline{R}(U) = \overline{R}(U) = U$

3) $\overline{R}(X\ \ Y) = \overline{R}(X)\ \ \overline{R}(Y)$

4) $\underline{R}(X\ \ Y) = \underline{R}(X)\ \ \underline{R}(Y)$

5) $\underline{R}(X\ \ Y) \supseteq \underline{R}(X)\ \ \underline{R}(Y)$

6) $\overline{R}(X\ \ Y) \subseteq \overline{R}(X)\ \ \overline{R}(Y)$

7) $X \subseteq Y \longrightarrow \underline{R}(X) \subseteq \underline{R}(Y)$

& $\overline{R}(X) \subseteq \overline{R}(Y)$

8) $\underline{R}(-X) = -\overline{R}(X)$

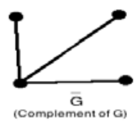
9) $\overline{R}(-X) = -\underline{R}(X)$

10) $\underline{R}\,\underline{R}(X) = \overline{R}\,\underline{R}(X) = \underline{R}(X)$

11) $\overline{R}\,\overline{R}(X) = \underline{R}\,\overline{R}(X) = \overline{R}(X)$

III.APPROXIMATIONS OF SUBGRAPH H OF G'

It was noticed that the graphs which are connected but the complement is open. See the simple example



Here we are defining the neighborhood relation for the complementary graph G' of G in [10] .

Definition 3.1.Let H be a subgraph of G' then the neighborhood of v is

$$N_C(v) = \{v\} \cup \{u \in V(G') : \overrightarrow{vu} \in E(G')\}.$$

Definition 3.2. For any sub graph H of G'. Then we define

(i) the lower approximation operation as follows:

$$\underline{V(H)} = \{v \in V(H): N_C(v) \subseteq V(H)\}.$$

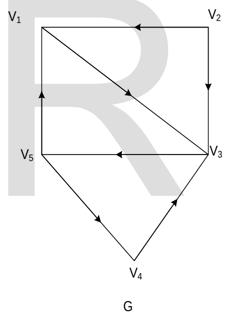
(ii) the upper approximation operation as follows:

$$\overline{V(H)} = \cup\ \{N_C(v): v \in V(H)\}.$$

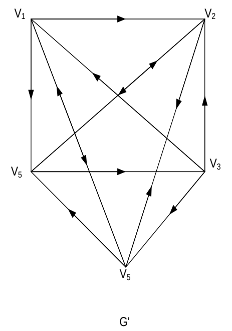
Definition3.3. An is defined as;

$$\mu(V(H)) = \frac{|\underline{V(H)}|}{|\overline{V(H)}|}$$, where $|\ |$ denotes the cardinality of the set and $V(H) \neq \phi$ such that $0 \leq \mu(V(H)) \leq 1$. If $\mu(V(H)) = 1$ then H is definable

Example 3.3. Given graph G in [10]



Complementary graph G' of G is as follows



For G', $V(G') = \{v_1,v_2,v_3,v_4,v_5\}$ and its neighbourhood are as follows:

$N_c(v_1) = \{v_1,v_2,v_4,v_5\}$,
$N_c(v_2) = \{v_2,v_4,v_5\}$,
$N_c(v_3) = \{v_1,v_2,v_3,v_4\}$,
$N_c(v_4) = \{v_1,v_2,v_4,v_5\}$ and
$N_c(v_5) = \{v_2,v_3,v_5\}$.

Accuracy measure of approximation of subgraph H of G' are calculated as follows:

$V(H)$	$\underline{V(H)}$	$\overline{V(H)}$	μ (V(H)
{v_1}	φ	**{ v_1,v_2,v_4,v_5 }**	**0**
{v_2}	φ	**{ v_2,v_4,v_5 }**	**0**
{v_3}	φ	**{ v_1,v_2,v_3, v_4,}**	**0**
{v_4}	φ	**{ v_1,v_2,v_4,v_5 }**	**0**
{v_5}	φ	**{ v_2,v_3,v_5}**	**0**
{ v_1,v_2}	φ	**{ v_1,v_2,v_4,v_5 }**	**0**
{ v_1,v_3}	φ	V(G')	**0**
{ v_1,v_4 }	φ	**{ v_1,v_2,v_4,v_5 }**	**0**
{ v_1,v_5 }	φ	V(G')	**0**
{ v_2,v_3}	φ	V(G')	**0**
{ v_2,v_4 }	φ	**{ v_1,v_2,v_4,v_5 }**	**0**
{ v_2,v_5 }	φ	**{ v_2,v_3,v_4, v_5}**	**0**
{ v_3,v_4}	φ	V(G')	**0**
{ v_3,v_5 }	φ	V(G')	**0**
{ v_4,v_5 }	φ	V(G')	**0**
{ v_1,v_2,v_3}	φ	V(G')	**0**
{ v_1,v_2,v_4 }	φ	**{ v_1,v_2,v_4,v_5 }**	**0**
{ v_1,v_2,v_5 }	φ	V(G')	**0**
{ v_1,v_3,v_4 }	φ	V(G')	**0**
{ v_1,v_3,v_5 }	φ	V(G')	**0**
{ v_1,v_4,v_5 }	φ	V(G')	**0**
{ v_2,v_3,v_4}	φ	V(G')	**0**
{ v_2,v_3,v_5 }	**{ v_5 }**	V(G')	**1/5**
{ v_2,v_4,v_5 }	**{ v_2 }**	V(G')	**1/5**
{ v_3,v_4,v_5 }	φ	V(G')	**0**
{ v_1,v_2,v_3,v_4 }	**{ v_3 }**	V(G')	**1/5**
{ v_1,v_2,v_3,v_5 }	**{ v_5 }**	V(G')	**1/5**
{ v_1,v_2,v_4,v_5 }	**{ v_1,v_2,v_4}**	V(G')	**3/5**
{ v_1,v_3,v_4,v_5 }	φ	V(G')	**0**
{ v_2,v_3,v_4,v_5 }	**{ v_2,v_5}**	V(G')	**2/5**
V(G')	V(G')	V(G')	**1**
Φ	φ	φ	**0**

Table1

IV.PROPERTIES OF APPROXIMATIONS OF A ROUGH SET FOR ANY SUBGRAPH H OF G' :

In this section properties of Rough set approximation for a subgraph H of G' are studied with examples taken from table **1.**

Proposition 1:. For any subgraph H of G' the lower approximation of any subgraph H of G' is a subset of H and H is a subgraph of the upper approximation.

i.e., $\underline{V(H)} \subseteq V(H) \subseteq \overline{V(H)}$

Example: V(H) = {v_2,v_3,v_5} ;

$\underline{V(H)}$ = {V_5}; $\overline{V(H)}$ = V(G')

Proposition 2:.The lower and upper approximation of a null graph are equal to itself and lower and upper approximation of G' are equal to the vertices of G' .

Example : If K is a null graph then

$\underline{K} = \varphi = \overline{K}$ and

and

$\underline{V}(G') = V(G') = \overline{V(G')}$.

Proposition 3: The upper approximation of union of any two subgraphs of G is equal to the union of upper approximation of those two subgraphs.

i.e., $\overline{V}(H \cup K) = \overline{V(H)} \cup \overline{V(K)}$.

Example : V(H) = { v_2,v_3,v_5 }; V(K) = { v_2,v_4,v_5 }

Proposition 4: The lower approximations of intersection of any two subgraphs are equal to the intersection of lower approximation of those two subgraphs.

i.e., $\underline{V}(H \cap K) = \underline{V(H)} \cap \underline{V(K)}$.

Example : V(H) = {$v_1,v_2.v_3$} ; V(K) = {v_1,v_2,v_5}

Proposition 5: The upper approximations of intersection of any two subgraphs is a subset of intersection of upper approximation of those two subgraphs.

i.e., $\overline{V}(H \cap K) \subseteq \overline{V(H)} \cap \overline{V(K)}$.

Example: V(H) = {v_2,v_4} ; V(K) = {v_2,v_5}

Proposition 6: If V(H) is a subset of V(K) then the upper approximation (lower approximation)of V(H) is contained in upper approximation(lower approximation) of V(K)

i.e If V(H) $\subseteq$ V(K) then $\overline{V(H)} \subseteq \overline{V(K)}$

and $\underline{V(H)} \subseteq \underline{V(K)}$.

Example: V(H) ={v_2,v_4,v_5}and V(K) ={v_2,v_3,v_4,v_5 }

Remarks: The following properties are not satisfied under lower and upper approximations of subgraph of G.

1. $\underline{V(-H)})$ need not be equal to $-\overline{V}(H)$, take $V(H)= \{ v_3,v_4\}$.
2. $\overline{V(-H)}$ need not be equal $-\underline{V(H)}$, take $V(H)= \{ v_1,v_2,v_5\}$.
3. $\overline{V}$ $(\overline{V(H)})$ need not be subset of $\overline{V(H)}$, take $V(H)= \{v_2,v_5\}$.
4. V(H) need not be a subset of $\underline{V}(\overline{V(H)})$, take $V(H)= \{ v_2,v_3,v_4,v_5\}$.
5. $\underline{V}(\overline{V(H)})$, need not be subset of $\underline{V(H)}$, take $V(H)= \{ v_2,v_4,v_5\}$.
6. V(H) need not be subset of $\overline{V}(\underline{V(H)})$, take $V(H)= \{ v_4,v_5\}$.
7. $\underline{V(H)}$ need not be subset of $\underline{V}(\underline{V(H)})$, take $\overline{V(H)}= \{v_5\}$.
8. $\overline{V}(\underline{V(H)})$ need not be a subset $\underline{V}$ (H), take $V(H)= \{ v_2,v_3,v_4,v_5\}$.

Note:

1. $\overline{V(H)}$ = V(G') if and only if for any vertex v∈ V(G)-V(H), there exist a vertex u∈ V(H) such that $\overrightarrow{uv}$ ∈ E(G').
2. Let G' = (V, $\overline{E}$) be a graph, H and K are two sub graphs of a graph G' . Then:
   a) $\underline{V}$(H ∪ K)) need not be subset of $\overline{V}$(H)∪$\overline{V}$(K), see Example 3.3, take $V(H)= \{ v_1,v_2,,v_4,v_5 \}$ and $V(K) = \{ v_1,v_2,v_3, v_5 \}$.
   b) $\overline{V(H)}$ ∩ $\overline{V(K)}$ need not be subset of $\overline{V}$(H ∩ K), see Example 3.3, take $V(H)=\{v_2,v_3,v_5\}$ and $V(K)=\{ v_2,v_4,v_5 \}$

The observation in G' is as follows:

Let G' = (V, $\overline{E}$) be a graph with non empty set of vertices then μ(V(G')) = 1.

Theorem 3.5. For any subgraph H of G' the following statement is true. For every vertex v∈ V(H) and any edge $\overrightarrow{vu}$ ∈ E(G') implies u ∈ V(H).

V. Conclusion and Future work:

In this paper computation of definable subgraph H of a complementary graph based on the approximations of rough set using neighborhood relation . By applying the concepts of accuracy measures of approximations of subgraph H of a complementary graph ,the calculations of exactness and roughness of a subgraph H of a complementary graph are to be made.The properties of rough sets also discussed for subgraphs of a complementary graph.In future,we can extend it for few more accuracy measures of definable subgraphs of a complementary graph like G' - accuracy, Semi G' accuracy and Pre G' -accuracy .

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